FUNCTION GRAPHS AND TRANSFORMATIONS

Program distributed by
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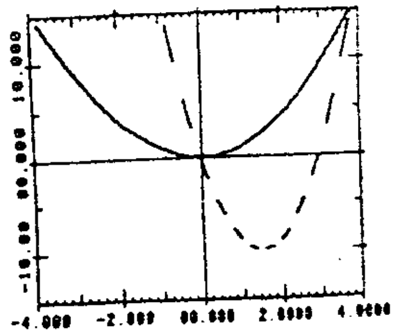

I. Introduction

A. Features.

This program uses the Apple II high resolution graphics capabilities to draw detailed graphs of functions which you define in Basic syntax. The graphs appear in a large rectangle whose edges are X and Y scales (with values labeled by up to 6 digits). The rectangle can be considered a "window" through which you look at the infinite two-dimensional plane; you can specify the position and size of this graphic window. Graphs can be superimposed, erased, and even drawn as dashed (rather than solid) curves. Two functions at a time are stored in the program, and various transformations can be applied to one of them: reflection, stretching or compressing (change of scale), and sliding (translation). Commands to the program are two letter abbreviations which are summarized in a "menu" at the top of the text display. The bottom of the text display is a scrolling window containing the alphanumeric input to and output from the program. You can switch back and forth at will between the text display and the graphic window. For your convenience in using the program while displaying the graphic window, the program's requests for commands or different types of parameters are prompted by different patterns of tones.

B. Purpose.

This program makes possible a visual, intuitive, and experimental approach to topics in algebra, trigonometry, and analytic geometry that many people find difficult to assimilate when presented in a primarily symbolic fashion. Perhaps more importantly, the program can be a vehicle for integrating the complementary intuitive and analytic/symbolic approaches; intuition and visualization, for example, can be used to help strengthen symbol manipulation skills.

C. Sample Applications.

1. To find solutions to the equation $f(x) = 0$, you can graph the function $y = f(x)$ and look for where it crosses the X axis. You can take successively closer looks at an X axis crossing (by specifying successively smaller windows), until you can read that solution from the graph to the accuracy you desire. (This graphic approach corresponds closely to widely used techniques for numerical solution of equations.)
2. To get experience in recognizing and transforming graphs and deriving formulas for them, you can have someone enter a "mystery" function, whose graph you will try to match. (The mystery function will probably be drawn as a dashed curve.) Your strategy will be
a) to decide what function or graph you know that is similar to the mystery function,
b) to enter its formula and graph it (see the illustration above), and then
c) to try to transform its graph so that it coincides with the graph of the mystery function. You may do this as a sequence of steps: for each step you will
i) choose and enter a transformation that will make the recently drawn graph more nearly the same as the graph of the mystery function,
ii) have the system erase the recent graph, and
iii) have the system graph the newly transformed function.
When the known function has been transformed into the mystery function and is then graphed, its graph will fill in the gaps in the graph of the mystery function (if the mystery function was originally drawn as a dashed curve). You can then derive the formula for the mystery function using the table of transformations

and their effects on formulas given later in this booklet.

Note: If graphs or their numerical labeling look strange, you may be using an early version of cassette or disk Applesoft II. See your Apple dealer to obtain a later version.

Note: For information about articles and workbooks relating to this program, contact the author:

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II. Graphing Commands.

GR: GRaph. Graph the current function. (The current function is
function 1 when the program is started. You can switch back and
forth between functions by using the commands F1 and F2.) The
graph is constructed of short line segments connecting points
whose coordinates have been calculated from the formula defin-
ing the function. (If any part of such a line segment would fall out-
side the graphic window, the segment is not drawn.) When the
first graph is drawn or if the window's position or size has been
redefined since the last graph was drawn, the screen will be
cleared and the window border will be traced out and labeled
before the graph is drawn. (If the window has not been redefined
since the last graph was drawn, the new graph will be superim-
posed on the current contents of the window.)
GD: Graph **D**ashed. Graph the current function as a dashed curve.
(Otherwise the same as GR.)
ER: ERase. Erase the most recently drawn graph.
EO: Erase **O**ld. Erase the graph drawn before the most recent one.
Note: a graph will not be erased by these commands if the current
graphing resolution (set by CR) is different from the graphing
resolution under which the graph was drawn.
CR: Change (graphing) **R**esolution. This changes (for succeeding
graphs) the number of points which are calculated, plotted and
connected to form the graph. (Specifically, the numerical
parameter for CR is the number of points for each of the eight ma-
jor horizontal divisions.) Relatively smooth functions can be
graphed well with a resolution of 5 or less. (5 is automatically the

initial resolution.) Note: do not attempt to change graphing resolution before graphing any functions; this wipes out the program (and cassette Applesoft).
XI: X-Interval. This command allows you to define the X interval which will be covered by the graphic window when the next graph is drawn. (The original X interval is −4 to 4.) Note: this command does not change the current graphic display.
YI: Y-Interval. This command allows you to define the Y interval which will be covered by the graphic window when the next graph is drawn. (The original Y interval is −15 to 15.) If you want the X and Y dimensions to have the same scale, the Y interval should be ¾ of the X interval; for example, the Y interval might be −3 to 3 when the X interval is −4 to 4.
YM: Y-int. Min. & Max. This command specifies that the Y interval should go from the minimum value (in the current X interval) of the function being graphed to the maximum value (in the current X interval). This is convenient if you don't have a precise idea of the range of function values in the X interval of interest. Note that this command causes the screen to be cleared before each graph is drawn; thus graphs cannot be superimposed when it is in effect.

Note that the labels for the Y scale (and the X scale) in the graphic window cannot have more than 5 or 6 digits (depending on the sign). If a label value is too large to fit in the allocated space, 6 dashes are printed in that space. However, the positions in the graphic window that are aligned with the unprintable label will still correspond to the value that couldn't be printed, and the graphing commands can still be used in a normal way.

III. Miscellaneous Commands.
TE: TExt. This causes the command menu and scrolling text window to be displayed.
Note: an attempt to graph or evaluate certain functions for certain values of X may produce Applesoft error messages like DIVISION BY ZERO, ILLEGAL QUANTITY, or OVERFLOW. If you hear the "blip" from Applesoft indicating this kind of error while you are viewing the graphic window, you must type TEXT (rather than just TE) to retrieve the text display and see what the error was.
GW: Graphic Window. This causes the current graphic window to be displayed.
ST: STop. This stops the program, adjusting the text display so

that the entire screen scrolls.
EV: EValuate. This instructs the system to evaluate the current function for the value of X you supply as the numerical parameter.
F1: Function **1**. This makes function 1 the current function. Commands such as the graphing or transformation commands will apply to it.

F2: Function **2**. This makes function 2 the current function. (Note that transformations cannot be applied to function 2.
CF: Change **F**unction definition. This command allows you to change the formula defining the current function. A function is defined with one or more Basic statements. (An example is given when the command is entered.) You should remember that Applesoft II has a non-standard precedence ordering: the function defined by the formula Y = — X^2 + 4X must be written for Applesoft II as Y = − (X^2) + 4* X, since negation has a higher precedence than exponentiation. Functions can be defined with more than one statement and even with more than one line. For example, the step function which is 0 when X<0, but is 1 when X> 0 could be defined as 250 Y = 0 : IF X> = 0 THEN Y = 1 Similarly, the top half of a circle of radius slightly more than 2 could be defined as 250 Y = 10^7 : IF X> = −2.00001 AND X<= 2.00001 THEN Y = SQR(4.00004 − X^2)
The purpose of the beginning of the definition is to give Y a value outside of the Y interval, so that nothing will be graphed when the absolute value of X is greater than the radius of the circle. The use of 2.00001 and 4.00004 rather than 2 and 4 is to avoid losing part of the graph due to truncation or roundoff error. Note that the bottom half of the circle can be obtained as a vertical reflection of the top half. (This works even when the slide transformations are in effect, because of the order in which the transformations are done.)

Note: Since the CF command actually stops the program, various kinds of data are stored to provide the context for the operation of the program after it has been restarted. The contents of the graphic window, of course, are saved, as are the current graphing resolution, which function is current, and the ends of the X and Y intervals. Other information is saved for possible use in erasing graphs, etc. Transformations are not saved, but are reset (even if you are changing the definition of function 2).

V. Transformation Commands

General Comments:

These transformations do not affect graphs already drawn. Their effects become apparent the next time this function 1 is graphed or evaluated. (The transformations can be applied only to function 1; if function 2 is current when a transformation is specified, an error message will be the only result.) The transformations are cumulative, and when function 1 is graphed or evaluated, the current transformations are always applied to the function in the order: reflect, stretch-compress, then slide. The transformations are reset to the identity transformations by the command RT and (as a side effect) by CF.

For discussions of transformations, see the references listed later. The table following this section gives the effects on formulas of the transactions available in this program. I belive that the table has been made more comprehensible by restricting the available transformations to ones which are purely horizontal or purley vertical, i.e., transformations which change the X coordinates of all (or almost all) points while changing the Y coordinates of none, or vice-versa.

Commands

RE: REflect. This reflects function 1 horizontally (about the Y axis) or vertically (about the X axis).

SC: Stretch-**C**ompress. This stretches or compresses function 1 horizonally or vertically by the amount specified. An amount between 0 and 1 gives a compression, while an amount greater than 1 gives a stretch. A vertical stretch-compress by a factor of 3, for example, triples all vertical distances but preserves horizontal distances. For a horizontal stretch or compress the Y axis is the fixed line (the set of points which are not moved by the transformation); for a vertical stretch or compress the X axis plays that role. The combinded effect of a horizontal stretch or compress and a vertical stretch or compress by the same amount is a size transformation which multiplies all distances by that amount and whose center or fixed point is the origin (since the origin is the intersection of the fixed lines of the two component transformations).

SL: SLide. This slides (or translates) function 1 horizontally or vertically by the amount specified. For horizontal slides a positive amount means a slide to the right and a negative amount means a slide to the left. Positive is up and negative is down for vertical slides.

RT: Reset Transformations. This resets the transformations to the identity transformations: no reflection, stretch-compress by a factor of 1, and slide by the amount of 0.

DT: Display Trasformations. This displays the current set of transformations. For each direction (horizontal and vertical) a single number represents both reflect and stretch-compress: a negative number indicates that there is a reflection in that direction, while a positive number indicates the absence of reflection, and the absolute value of the number gives the amount of stretching or compression.

Some Geometric Transforamtions

Transformation Name		Its Parameters	Its Effect on a Single Point	How To Compensate For its Effect in the Formula for a Relation
REFLECT		Vertically (about the X axis)	Moves (x, y) to (x − y)	Replace all occurrences of y with − y
		Horizontally (about the Y axis)	(x,y) ⟶ (− x,y)	Replace x with − x
SLIDE (OR TRANSLATE)		Vertically by an amount k (k>0 is a slide up; k>0 is a slide down)	(x,y) ⟶ (x,y + k)	Replace y with y - k
		Horizontally by an amount h (h>0 is a slide right; h>0 is a slide left)	(x,y) ⟶ (x + h,y)	Replace x with x − h
STRETCH-COMPRESS		Vertically (with the X axis as the fixed line) by a factor of b (b>1 is a stretch; 0>b>1 is a compress)	(x,y) ⟶ (x,by)	Replace y with y/b
		Horizontally (with the Y axis as the fixed line) by a factor of a (a>1 is a stretch; 0>a>1 is a compress	(x,y) ⟶ (ax,y)	Replace x with x/a

Example of the use of Transformations:

Suppose you consider the original function designated by F2 (the one which comes as part of the program) as a mystery function and attempt to discover its formula graphically, following the pattern given in section C.2. of the introduction. You start by graphing it as a dashed curve and then choose $Y = X^2$ as a simple function likely to be similar to the mystery function. Entering $Y = X^2$ as F1 and graphing it, you get a picture something like the one at the beginning of this documentation, except that the two function graphs have the same shape (and orientation) and differ only in placement. A horizontial slide and a vertical slide are necessary to transform the known function $Y = X^2$ into the mystery function. By sliding the known function 1.5 units to the right you align the graphs horizontally; the formula $Y = (X - 1.5)^2$ represents the transformed function. Suppose you now slide vertically by the amount -2. When you graph the result, you see that you have not yet matched the mystery function. (To discover exactly what transformation is needed , it may help to change the Y interval to go from -3 to 0 and then to redraw the two graphs.) If you now slide down another .25 units, the graph of the result coincides with the graph of the mystery function, filling in the gaps in its dashed graph. The net vertical slide is -2.25 (you can confirm this by using the command DT to display the transformations currently in effect), so the final formula for the transformed function is

$$Y - (-2.25) = (X - 1.5)^2$$
$$\text{or } Y + 2.25 = X^2 - 3X + 2.25$$
$$\text{or } Y = X^2 - 3X$$

If you list line 350 of the program, you will see that this is indeed the formula for the mystery function.

V. References.

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Dodge, Clayton W. (1972), *Euclidean Geometry and Transformations*. Reading, Mass.: Addison-Wesley.

Gans, David (1969), *Transformations and Geometries.* New York: Appleton-Century-Crofts.

Miller, Charles D. and Vern Heeren (1973), *Mathematical Ideas: An introduction,* Second Edition. Glenview, Ill.: Scott, Foresman.

Sobel, Max A. and Norbert Lerner (1979), *Algebra and Trigonometry: A Pre-Calculus Approach.* Englewood Cliffs, N.J.: Prentice Hall.

Stone, Don (1976), "Developing Mathematical Intuition through Graphing and Working with Functions on an Alphanumeric Video Display Terminal." *Proceedings of the Third Annual New Jersey Conference on the Use of Computers in Higher Education,* March 22 & 23, 1976, Rutgers University. (Reprints available from the author.)

Usiskin, Zalman (1976), *Advanced Algebra with Transformations and Applications. River Forest, Ill.: Laidlaw Brothers. (A high school textbook with an approach closely related to the philosophy underlying this program.)*

LOADING INSTRUCTIONS

1. Press "RESET" key. (asterisk and cursor appear)
2. Hold the "CTRL" key down and press the "B" key. Then press "RETURN".
3. If working with DOS type In FP to load in Applesoft II basic, otherwise load Applesoft II basic from cassette.
4. Type "LOAD" and press "RETURN" and start tape recorder.
5. When the cursor reappears type "RUN" and press "RETURN"

Note: If you wish to speed up the introductory information displayed at the beginning of the program type "RUN 8900" instead of "RUN".

GUARANTEE

POWERSOFT, INC. guarantees the playback of its pre-recorded tapes when purchased new, provided the playback head of the tape recorder used is properly aligned. All pre-recorded tapes are produced on the finest quality professional duplicating equipment available. *The program is recorded at least twice on the cassette.*